Mathematical Modeling of COVID-19 Transmission by a k Phases SEIR Model

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Mathematical Modeling of COVID-19 Transmission by a k Phases SEIR Model

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Background

- Mathematical models can provide insights on the spread of infectious diseases, such as the novel SARS-CoV-2 (COVID-19).
  - This work applied a SEIR epidemiological compartmental model (susceptible-exposed-infected-recovered) with k phases to predict the actual spread of the COVID-19 virus.

Methods

- During the mathematical modeling of the COVID-19 spread, four parameters of the SEIR model (Fig. 1) were obtained by international experiences: the incubation period = 3.7 in days, the proportion of critical cases = 0.05, the overall case-fatality rate = 0.023, and the asymptomatic proportion of COVID-19 = 0.18.
- These values can be modified for a specific region, but, the critical step in the prediction of COVID-19 by the model is the value of R0 (the basic reproduction number) and T_infectious (the infectious period, in days).
- To solve this difficult, R0 and T_infectious were calculated by mathematical constrained optimization, a numerical method. Differently from a statistical modelling, a numerical method is a type of mathematical modelling that is not dependent on a probability distribution.
- In this numerical modelling process the objective function that measures the model error is minimized with respect to R0 and T_infectious in the presence of constraints on those variables. For R0, constraints are valid range of values (0.5 ≤ R0 ≤ 20). For T_infectious (days), constraints also are related to its range of values (2 ≤ T_infectious ≤ 14).
- A Solver from Excel or NEOS Server, for example can be used for finding numerically minimum of a function Z, that represents the sum of squared errors between each COVID-19 new case observed in one day, and the COVID-19 cases predicted by the SEIR model (Fig. 2).

Fig 1 - The SEIR model for COVID-19.

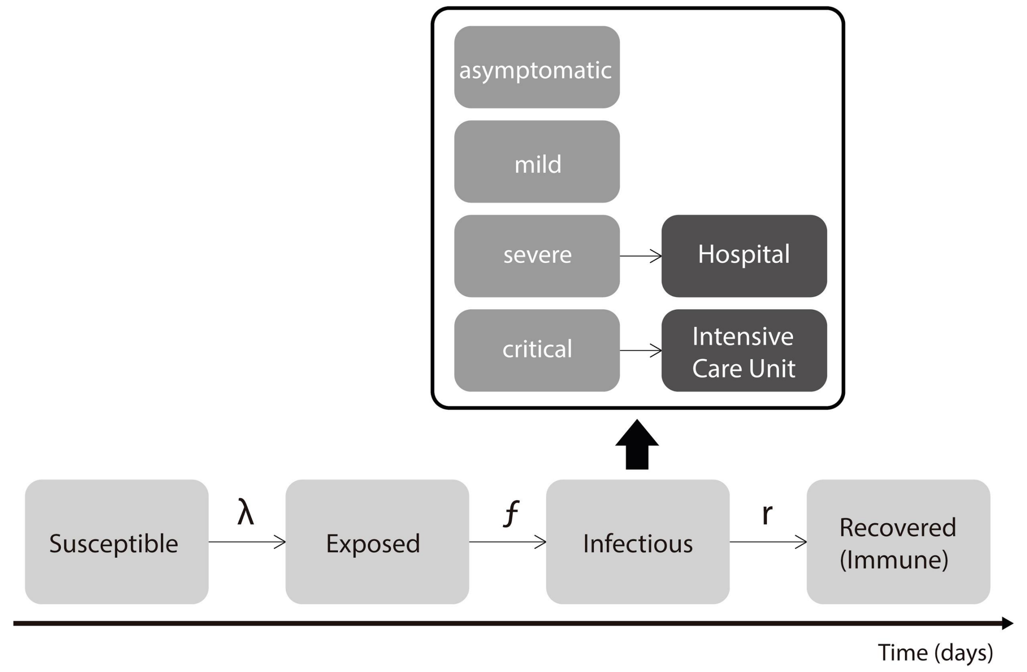

Fig 2 - Algorithm for the SEIR model applied to COVID-19: initialization and calculation of new COVID-19 cases day-by-day.

```
Algorithm COVID
   Read N {population's size}
   Read T_phase_I {duration of the first phase of the epidemic (days)}
   Read = R0_phase_I {basic reproduction number for COVID-19 from the first phase of the epidemic}
   Read T_infectious_phase_I {the infectious period from the first phase of the epidemic (days)}
   Read T_phase_II {duration of the second phase of the epidemic (days)}
   Read = R0_phase_II {basic reproduction number for COVID-19 from the second phase of the epidemic}
   Read T_infectious_phase_II {the infectious period from the second phase of the epidemic (days)}
   Read = R0_phase_III {basic reproduction number for COVID-19 from the third phase of the epidemic}
   Read T_infectious_phase_III {the infectious period from the thrid phase of the epidemic (days)}
   T_incubacao = 3.7 {COVID-19 the incubation period, in days}
   p_CTI = 0.05 {spectrum of disease: proportion of critical COVID-19 cases}
   letalidade = 0.023 {the overall case-fatality rate}
   p_assintomaticos = 0.18 {asymptomatic proportion of COVID-19}
   f = 1/T_incubacao {the rate at which individuals move from the latent class to the infected class}
   t = 0 {first day = "zero day"}
   t_Max = 180 {last day of simulation: 180 days after the first COVID-19 case}
   Susceptivel[t] = (N - COVID) {susceptible individuals}
   Preinfec[t] = 0 {exposed or latent patients}
   COVID[t] = 1 {infected patients or COVID-19 cases}
   Imunes[t] = 0 {recovered or immune patients}
   CTI[t] = 0 {critical cases}
   Obitos[t] = 0 {case-fatality}

   Repeat
      t = t + 1
      If (t ≤ T_phase_I)then
                     ecr = R0_phase_I/T_infectious_phase_I {effective contact rate for phase 1}
                     recupera = 1/T_infectious_pahse_I {recovery rate for phase 1}
                else Se (t ≤ T_phase_II)then
                     ecr = R0_phase_II/T_infectious_phase_II {effective contact rate for phase 2}
                     recupera = 1/T_infectious_pahse_II {recovery rate for phase 2}
                else
                     ecr = R0_phase_III/T_infectious_phase_III {contact rate for phase 3}
                     recupera = 1/T_infectious_pahse_III {recovery rate for phase 3}
      End If

      Beta = ecr/N {transmission rate}
      Susceptivel[t] = Susceptivel[t-1] - Beta*COVID[t-1]*Susceptivel[t-1]
      Preinfec[t] = Preinfec[t-1] + Beta*COVID[t-1]*Susceptivel[t-1]-f*Preinfec[t-1]
      COVID[t] = COVID[t-1] + f*Preinfec[t-1] - recupera*COVID[t-1]
      Imunes[t] = Imuntes[t-1] + recupera*COVID[t-1]
      CTI[t] = p_CTI*COVID[t]
      Obitos[t] = letalidade*Imunes[t]*p_assintomaticos
      Write t, Susceptivel[t],Preinfec[t],COVID[t],Imunes[t],CTI[t],OBitos[t]
      If (t = t_Max)
          Then break
      End If
   End Repeat
End Algorithm
```

Results

- Based on real COVID-19 data until April 11, R0 and the infectious periods were calculated for the third phase of the epidemic by the mathematical constrained optimization used for finding the numerically minimum of a function Z (Fig. 3) for 15 countries (Fig. 4).

Conclusion

- The k phases SEIR model proved to be a useful to measure the COVID-19 transmission in a City, State or Country. A four-phase model can be applied to fit a scenario with a new second COVID wave.

Fig 3 - Mathematical constrained optimization model.

$$Minimize\ Z = \sum_{i=1}^{D} |I_i - \hat{I}_i| \quad (1)$$

Subject to
- T_phase_I ≤ D
- 0.5 ≤ R0_phase_I ≤ 20
- 2 ≤ T_infectious_phase_I ≤ 14
- T_phase_II ≤ T_phase_I
- 0.5 ≤ R0_phase_II ≤ 20
- 2 ≤ T_infectious_phase_II ≤ 14
- 0.5 ≤ R0_phase_III ≤ 20
- 2 ≤ T_infectious_phase_III ≤ 14

Where:
- I_i = number of COVID-19 new cases observed in a city, state or country during the day i;
- $\hat{I}_i$ = number of COVID-19 new cases predicted by the model in the day i;
- D = total days of the epidemic in a city, state or country;
- T_phase_I = duration of the first phase of the epidemic (days);
- R0_phase_I = basic reproduction number for COVID-19 from the first phase of the epidemic;
- T_infectious_phase_I = the infectious period from the first phase of the epidemic (days);
- T_phase_II = duration of the second phase of the epidemic (days);
- R0_phase_II = basic reproduction number for COVID-19 from the second phase of the epidemic;
- T_infectious_phase_II = the infectious period from the second phase of the epidemic (days);
- R0_phase_III = basic reproduction number for COVID-19 from the third phase of the epidemic;
- T_infectious_phase_III = the infectious period from the third phase of the epidemic (days);

Fig 4 - Mathematical Modeling of COVID-19 Transmission by a 3 Phases SEIR Model in 15 countries.